2021

PHYSICS — HONOURS

Paper : CC-11

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Syllabus : 2019-2020

(Electromagnetic Theory)

Answer ***question no.*** **1** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) Calculate the intrinsic impedance for a plane wave in the linear isotropic medium for which relative permittivity and relative permeability are 3 and 2 respectively.

(b) Determine the mean value of the magnetic field in air at a distance of 250 cm from a radiating source of power 1 kW.

(c) Sea water at a frequency 400 MHz has relative permittivity 81. If the resistivity is 0.23Ωm, find the ratio of conduction current to displacement current.

(d) Consider an infinitely long solenoid with N turns per unit length, radius R and carrying a current $I(t) = I_0 \cos\omega t$. Derive the electric field at the surface of the solenoid.

(e) What should be the angle of the Sun above horizon, so that sunlight reflected from a still lake is plane polarized? Take refractive index of water is 1.33.

(f) Define optic axis and principal section of a crystal.

(g) Describe the state of polarization of wave represented by the following set of equations :

$$E_x = E_0 \cos(kz - \omega t) \text{ and } E_y = E_0 \cos(kz - \omega t + \pi/4).$$

2. (a) Show that the electric field $\vec{E}$, magnetic field $\vec{B}$ and the propagation unit vector $\hat{n}$ are related by $c\vec{B} = \hat{n} \times \vec{E}$. How do the fields vary with distance long away from the source?

(b) Suppose there are two sets of vector and scalar potentials, $(\vec{A}, \varphi)$ and $(\vec{A}', \varphi')$. By how much the two potentials differ so that Electric field $\vec{E}$ and magnetic field $\vec{B}$ remain invariant? Justify your answer. What is the importance of Lorentz gauge?

(c) If vector potential $\vec{A} = \beta x\hat{i} + 2y\hat{j} - 3z\hat{k}$ satisfies the Coulomb gauge condition, what is the value of β? 3+(3+2)+2

3. Consider two electromagnetic waves propagating in Vacuum with their electric field vectors $\vec{E_1} = E_0 \cos(kz - \omega t)\hat{i}$ and $\vec{E_2} = E_0 \cos(kz + \omega t)\hat{i}$.

(a) Evaluate the magnetic field $\vec{B}$ corresponding to the superposition of these two waves.

(b) Calculate time-averaged energy density $\langle u \rangle$.

(c) Calculate Poynting vector $\vec{S}$.

(d) Check whether $\frac{\partial u}{\partial t} + \vec{\nabla}.\vec{S} = 0$. 2+2+3+3

4. (a) Starting from Maxwell's equations, show that any initial charge density in a conductor dissipates in a characteristic time.

(b) A beam of light of frequency ω is reflected from a dielectric-metal interface at normal incidence. The refractive index of the dielectric medium is $n_1 = n$ and that of the metal is $n_2 = n(1 + i\rho)$. If the beam is polarized parallel to the interface, calculate the phase change experienced by the light upon reflection.

(c) Derive the expression for skin depth in a conductor. An EM shielded room is designed so that at a frequency $\omega = 10^7$ rad/s the intensity of the external radiation that penetrates the room is 1% of the incident radiation. If $\sigma = (1/2\pi) \times 10^6\ (\Omega\text{m})^{-1}$ is the conductivity of the shielding material, calculate its minimum thickness. 2+3+(3+2)

5. (a) Two infinitely extended homogeneous isotropic dielectric media (medium 1 and 2 with dielectric constants $\varepsilon_1/\varepsilon_0 = 2$ and $\varepsilon_2/\varepsilon_0 = 5$, respectively) meet at the $z = 0$ plane. A uniform electric field exists everywhere. For $z > 0$ plane, the electric field is given by $\bar{E}_1 = 2\hat{i} - 3\hat{j} + 5\hat{k}$. The interface separating the two media is charge free. Determine the electric displacement vector in the medium 2.

(b) An electromagnetic wave is incident at the surface of two linear homogeneous dielectrics. Find out the ratio of the electric field intensities for normal incidence. Find out the conditions under which there is a phase reversal for reflected wave.

(c) A plane electromagnetic wave falls obliquely on air-glass interface. Find the angle of incidence for which the reflection and transmission coefficients are each equal to 0.3. $(n_{\text{glass}} = 1.5)$

2+(3+2)+3

6. (a) Explain the phenomenon of double refraction in a uniaxial crystal by applying Huygen's theory.

(b) A left circular light of wavelength 656nm is to be converted to right circular by passing through a quartz retarder $(n_e = 1.551, n_o = 1.542)$. Compute the minimum thickness of the retarder.

(c) A thin polaroid, placed between two crossed polaroids is allowed to rotate at a rate 'ω' about their common central axis. Determine the intensity of transmitted light in terms of intensity of unpolarized light.

(d) Calculate the Brewster's angle for glass to air refraction. Refractive index of glass is 1.5.

3+2+3+2

7. (a) A ray of yellow light (λ = 5893A°) incident on a doubly refracting plate at an angle 50°. The plate is cut so that the optic axis is perpendicular to the plane of incidence and parallel to the front face. Find the angular separation between two emerging rays. (Given n_o = 1.662, n_e = 1.474)

(b) State Biot's law of rotatory polarization.

(c) A solution of concentration 6.0 gm/100 cc used in a tube of length 33cm causes 14°30′ rotation in the plane of polarization of light of wavelength 550nm. Deduce the specific rotation. Also estimate the rotation it would cause for wavelength 450nm. 4+2+(2+2)

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Syllabus : 2018-2019

(Quantum Mechanics and Applications)

Full Marks : 50

Answer ***question no.*** **1** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) Show that the variance of Hamiltonian in a stationary state is zero.

(b) The wave function of a particle is given by

$$\psi(x) = A\left(a^2 - x^2\right) \quad \text{for } |x| \le a$$
$$= 0 \quad \text{otherwise}$$

with $A = \sqrt{15/\left(16a^5\right)}$. Calculate the expectation value of x^2.

(c) Prove that, if $\hat{A}$, and $\hat{B}$ are two Hermitian operators, then their commutator $\left[\hat{A}, \hat{B}\right]$ is an anti-Hermitian operator.

(d) Find the eigenvalues and eigenfunctions of the angular momentum operator $\hat{L}_z = i\hbar\frac{\partial}{\partial\phi}$.

(e) The electronic configuration of Mg is $1s^2 2s^2 2p^6 3s^2$. Obtain its spectral term.

(f) How Paschen-Back effect is different from Zeeman effect?

(g) What are the L, S and J values for the state $^5F_5$?

2. (a) Find the eigenfunction of the operator $\frac{df}{dx} + \frac{d}{dx}$ with eigenvalue λ where $f(x)$ is an analytic function of x.

(b) Evaluate the commutator $[x^n, p_x]$ where n is a positive integer.

(c) Consider a particle confined in a one-dimensional box extending over the region $-L/2 < x < L/2$. The particle is in ground state. The box is expanded suddenly to $-L < x < L$ leaving the wave function momentarily undisturbed and the energy is measured. Calculate the probability that the result will be the same as the ground state energy of the expanded box. 3+2+5

3. (a) Let $\sigma = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

Prove that for any scalar x, $\exp(x\sigma) = \cosh x\underline{1} + \sinh x\sigma\underline{1}$ where $\underline{1}$ is the 2×2 identity matrix.

(b) A one-dimensional simple harmonic oscillator of angular frequency ω has a normalized wave function

$$|\psi\rangle = \frac{1}{\sqrt{3}}|2\rangle + c|7\rangle$$

where $|n\rangle$ are normalized energy eigenstates. Find the value of the constant c. Hence find the expectation value of energy in the state $|\psi\rangle$.

(c) A charged particle with mass m and charge q is moving in a one-dimensional harmonic potential of angular frequency ω. In addition, it is also subjected to a constant external electric field ε. Write down the Hamiltonian of the particle and find the energy eigenvalues. 2+4+4

4. (a) Evaluate the commutator $[x, L_z]$.

(b) For a spin $\frac{1}{2}$ particle let us choose the eigenstates of S_z with eigenvalues $\pm\hbar/2$ as the basis states.

If a particle is in state $|\psi\rangle = \begin{pmatrix} \cos\alpha \\ e^{i\beta}\sin\alpha \end{pmatrix}$

where α, β are real constants, what is the probability that a measurement of S_y on $|\psi\rangle$ will give $(-\hbar/2)$?

(c) Consider a particle with wavefunction $\psi(x, y, z) = Az\exp\left[-b\left(x^2 + y^2 + z^2\right)\right]$

where A and b are constants. Show that this wavefunction is an eigenstate of the operators L^2 and L_z and find the corresponding eigenvalues. 2+4+4

5. (a) The normalised wavefunction for the ground state of hydrogen atom is $\psi_{100} = \frac{1}{\sqrt{\pi a^3}}e^{-r/a}$

where 'a' is the Bohr radius. Calculate (i) the most probable distance of the electron from the nucleus and (ii) the average kinetic energy.

(b) For a particle in a box $0 \le x \le L$, the wavefunction is given by $\psi(x) = A\sqrt{x(L-x)}$.

Calculate A from the normalisation condition. Find the probability that the particle will be found in the inverval $0 \le x \le (L/4)$.

(c) Let $\hat{p}$ be the momentum operator and let $f(x)$ be a smooth function of position x. Prove that

$$\exp\left(\frac{i\hat{p}a}{\hbar}\right)f(x) = f(x+a)$$

where a is a real constant. 4+4+2

6. (a) What is the origin of fine structure for a hydrogen-like atom? How the Lyman α line is split due to fine structure?

(b) The limit for Balmer series of hydrogen atom is 3646 Å. Calculate the atomic number of the element which gives X-ray wavelength down to 10 Å.

(c) The spacing between the vibrational levels of CO molecule is 0.08 eV. Calculate the value of the force constant of the CO bond. Given that the masses of C and O atom are 2.0×10^{-26} kg and 2.7×10^{-26} kg respectively. ($\hbar = 6.58 \times 10^{-16}$ eV sec). (1+3)+3+3

7. (a) Calculate the Landé g factor for the two levels participating in sodium D_1 line.

(b) The sodium D_1 line emitted from a particular region of the sun is found to be split into four components in anomalous Zeeman effect. What is the strength of the solar magnetic field B in that region if the wavelength difference between the shortest and the longest wavelengths is 0.022 nm? (The wavelength of the D_1 line is 589.8 nm, $\mu_B/hc = 46.65$ T^{-1} m^{-1}.) 4+6